THE TASK OF OPTIMIZING REGIONAL DEPLOYMENT OF COMMERCIAL DRONE BASES

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Abstract: *The authors set the task of optimizing deployment of commercial drone bases as part of the mobile units of a commercial drone company providing services to clients within a designated territory. Main commercial services include monitoring of local infrastructure and agricultural areas, advertising/promotional services, events, excursion tourism. The efficiency criterion and initial conditions are selected. The task is formulated as a typical problem of regional economics. A linear programming method is offered to solve the task. The method is validated by a numerical example.*

Keywords: *drones, mobile unit, commercial use, basing (deployment) optimization task.*

JEL classification: *O32*

1 INTRODUCTION

The authors review the task of optimizing the regional deployment of drone bases for the mobile units (MU) of commercial drone companies (DCo). Apart from drones, a MU also incorporates: crews (pilots, operators, drivers of specially equipped motor cars) and universal means of support – means of remote delivery of drones to the points of use (those are, normally, motor cars, but also motor boats, etc.)

Most of the clients of a DCo order video monitoring services for the status of various types of facilities (infrastructure, agriculture, real estate, etc.) or events (ceremonial events, celebrations, etc.) in online mode with possible subsequent office-based processing of the monitoring results and presenting them in the form of video commercials, advertising booklets, etc.

The task of optimizing MU basing (deployment) must be set and solved for a DCo in view of its operational and customer service efficiency.

Classically, the task is a typical problem of regional economics related to the choice of the place for deployment of production facilities, such as mining plants, agricultural products processing plants, bank branches, etc. (Krarup J., Pruzan P.M.,1983; Cornuejols G., Fisher M.L., 1977; Mirchandani P.B., Francis R.L., 1990) A comprehensive review of deployment criteria for the aforesaid facilities is given in (Reza Zanjirani Farahani, Maryam Steadie Seifi, NasrinAsgari,2010). An option of setting the task (but not solving it) with respect to a DCo is presented in (Sulima N., 2017).

The distinctive features of the reviewed task and its reference conditions, unlike those mentioned above, are as follows:

- the application sphere, which has not been reviewed in scientific research literature yet;
- the subject of sale is services, whose provision on certain territories is more dependant on force majeure circumstances (weather conditions, natural disasters, etc.);
- the services are provided at the point of their generation (not the goods are delivered to a company but rather the company is 'transported' to the point of service provision);
- the services may only be provided during daylight hours.

2 MAIN PART

2.1 Setting the Task

As the main efficiency criterion of MU basing, we can choose the minimum expenditure of the DCo providing a specified set of services to the clients within designated areas of the region. Such setting presumes that the revenue side of the DCo will remain unchanged, while the expenditure will depend, mostly, on the number and specific location of MU bases, and territorial location of the clients' facilities. Assuming there is one MU per base, the more MUs - the higher are the costs of operation, and the less time is required to complete the clients' orders within a relevant territory, and the more orders may be accomplished, and vice versa.

Thus, let us formulate the task of optimizing MU basing (deployment) as a Boolean programming problem:

To minimize the function

$$\min\ F(x,z) = \sum_{i=1}^{n} f_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{m} c_{i,j} z_{i,j} \qquad (1)$$

by Boolean variables $x_i \in X$, $z_{i,j} \in Z$ given the following restrictions:

$$\sum_{i=1}^{n} x_i \le k, \tag{2}$$

$$x_i \le 1, \quad i = 1,\ldots,n, \tag{3}$$

$$\sum_{i=1}^{n} z_{i,j} = 1, \quad j = 1,\ldots,m, \tag{4}$$

$$\sum_{j=1}^{m} z_{i,j} \le s_i x_i, i = 1,\ldots,n. \tag{5}$$

The initial data of the set task:

I – the multitude of possible points of deployment, the total number being $n = |I|$;

J – the multitude of points of rendering drone services (assuming, for the sake of simplicity, that it is equivalent to the multitude of the clients served), $m = |J|$;

f_i – the costs of organization and maintenance of a base at Point $i \in I$;

$c_{i,j}$ – costs as at the assignment of Client j to Base i, $j \in J$, which are determined, mostly, by the cost of transportation to the point of service provision;

k – the maximum possible number of deployment points (bases),

s_i – the maximum number of clients assigned to Base i;

The sought variables are as follows:

k^*– the optimal number of bases;

$x_i = 1$ if a base is opened at Point i, and 0 - if otherwise;

$z_{i,j} = 1$ if a base at Point i renders services to Client j.

The above described problem has $2n + m + 1$ restrictions and $n(m + 1)$ variables. To solve the problem, we suggest using the linear programming method (Andronovs A.,2007; Nocedal J., Wright S.J.,2006).

2.2 The Specifics of the Reviewed Linear Programming Problem

An important requirement of the above formulated problem is that unknown variables may only have either of two values: 0 and 1. It makes the problem-solving process much more complicated. The known methods of solving are iterative and presume the existence of a certain initial solution (Andronovs A., 2007). Unless the integrality condition is absent, we have a linear programming problem with a target function (1), restrictions (2) - (5) and nonnegative variables $\{x_i, z_{i,j} : i = 1,\ldots,n; j = 1,\ldots,m\}$.

Furthermore, *diag*(*s*) designates a diagonal matrix with vector *s* as a main diagonal.

If we introduce 2*n* + 1 complementary variables v_i, u_i and *t*, we will get the following equations in restrictions (2), (3) and (5):

$$\sum_{i=1}^{n} x_i + t = k, \tag{6}$$

$$x_i + v_i = 1, \quad i = 1,\ldots,n, \tag{7}$$

$$\sum_{j=1}^{m} z_{i,j} - s_i x_i + u_i = 0, \quad i = 1,\ldots,n. \tag{8}$$

Consider the following vectors:

$x = (x_1,\ldots,x_n)^T$ is an *n*-dimensional column-vector;

$s = (s_1,\ldots, s_n)^T$ is an *n*-dimensional column-vector;

$u = (u_1,\ldots,u_n)^T$ is an *n*-dimensional column-vector;

$v = (v_1,\ldots,v_n)^T$ is an *n*-dimensional column-vector;

$z_{,j} = (z_{1,j},\ldots, z_{n,j})^T$ is an *n*-dimensional column-vector;

$z = (z_1^T,\ldots, z_m^T)^T = (z_{1,1},\ldots, z_{n,1}, z_{1,2},\ldots, z_{n,2},\ldots, z_{1,m},\ldots, z_{n,m})^T$ is an *nm*-dimensional column-vector;

$w = (x^T, z^T, u^T, v^T, t)^T$ is a $(3n + nm + 1)$-dimensional column-vector;

$row(n) = (1, \ldots, 1)$ is an *n*-dimensional row-vector of units,

$zero(m) = (0, \ldots, 0)$ is an *m*-dimensional row-vector of zeros,

$f = (f_1,\ldots f_n)^T$ is an *n*-dimensional column-vector of cost coefficients with $\{x_i\}$;

$c = (c_{1,1},\ldots, c_{n,1}, c_{1,2},\ldots, row(n))^T$ is an *nm*-dimensional column-vector of cost coefficients with $\{z_{i,j}\}$;

$d = (k, row(n), row(m), zero(n))^T$ is a $(2n + m + 1)$-dimensional column-vector of free terms.

Next, we should introduce some designations for matrices. Let us assume that $O_{m\times n}$ means a matrix of the order of $m\times n$ of zeros. Now we will introduce the Kronecker product of matrices. Assuming Matrix *A* has the order of $m\times n$, and Matrix *B* – the order of $k\times p$, then their Kronecker product $A \otimes B$ is a matrix of the order of $mk \times np$ resultant from replacing, in matrix A, of each element $A_{i,j}$ with matrix B multiplied by $A_{i,j}$.

So, let us construct restriction matrix A of the linear programming problem. It will be a block matrix of the order of $(2n + m + 1) \times (3n + nm + 1)$ as follows:

$$A = \begin{pmatrix} A_{0,1} & A_{0,2} & A_{0,3} & A_{0,4} \\ A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \end{pmatrix},$$

where

$$A_{0,1} = row(n) = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}_{1\times n},$$
$$A_{0,2} = zero(nm) = \begin{pmatrix} 0 & \dots & 0 \end{pmatrix}_{1\times nm},$$
$$A_{0,3} = zero(n) = \begin{pmatrix} 0 & \dots & 0 \end{pmatrix}_{1\times n},$$
$$A_{0,4} = zero(n) = \begin{pmatrix} 0 & \dots & 0 & 1 \end{pmatrix}_{1\times (n+1)},$$
$$A_{1,1} = I(n),$$
$$A_{1,2} = O_{n\times mn},$$
$$A_{1,3} = O_{n\times (n}$$
$$A_{1,4} = \left(I(n) \middle| zero(n)^T \right),$$
$$A_{2,1} = O_{m\times n},$$
$$A_{2,2} = I(m) \otimes row(n)_{m\times nm},$$
$$A_{2,3} = O_{m\times n},$$
$$A_{2,4} = O_{m\times (n+1)},$$
$$A_{3,1} = -diag(s),$$
$$A_{3,2} = row(m) \otimes I(n)_{n\times nm},$$
$$A_{3,3} = I(n),$$
$$A_{3,4} = O_{n\times (n+1)},$$

Now the linear programming problem is formulated as follows:

To minimize the function

$$F(w) = F(x, z, u, v, t) = f^T x + c^T z \tag{9}$$

By nonnegative variables x_i, $z_{i,j}$, v_i, u_i, t with the following restrictions:

$$Aw = d. \tag{10}$$

Let us assume that $\tilde{v} = \begin{pmatrix} v^T & t \end{pmatrix}^T = \begin{pmatrix} v_1 & \ldots & v_n & t \end{pmatrix}^T$. Then all the restrictions may be presented in expanded form as follows:

$$A = \begin{pmatrix} A_{0,1} & A_{0,2} & A_{0,3} & A_{0,4} \\ A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \end{pmatrix} \begin{pmatrix} x \\ z \\ u \\ \tilde{v} \end{pmatrix} = \begin{pmatrix} k \\ row(n) \\ row(m) \\ zero(n) \end{pmatrix},$$

$$\begin{array}{ccccccccc} A_{0,1}x & + & A_{0,2}z & + & A_{0,3}u & + & A_{0,4}\tilde{v} & = & k, \\ A_{1,1}x & + & A_{1,2}z & + & A_{1,3}u & + & A_{1,4}\tilde{v} & = & row(n), \\ A_{2,1}x & + & A_{2,2}z & + & A_{2,3}u & + & A_{2,4}\tilde{v} & = & row(m), \\ A_{3,1}x & + & A_{3,2}z & + & A_{3,3}u & + & A_{3,4}\tilde{v} & = & zero(n), \end{array}$$

$$\begin{array}{ccccccc} row(n)x & + & & & t & = & k, \\ I(n)x & + & & & \left(I(n) \middle| zero(n)^T\right)\tilde{v} & = & row(n), \\ & & I(m)\otimes row(n)z & & & = & row(m), \\ -diag(s)x & + & row(m)\otimes I(n)\ z & + \ I(n)u & 0 & = & zero(n), \end{array}$$

Now the problem has $2n + m + 1$ restrictions and $n(m + 3) + 1$ variables.

2.3 The Specifics of Computational Procedure for the Reviewed Linear Programming Problem

To solve the problem (9), (10) we may use the known methods of linear programming, e.g., the simplex method, the modified simplex method, etc. (Andronovs A., 2007). Of all the $n(m + 3) + 1$ unknown variables, $\theta = 2n + m + 1$ variables are named as *basic* – by the number of restrictions. The other, non-basic variables equal to zero. Each variable correlates to the θ–dimensional column-vector of matrix A. If the matrix, which is made up of the columns of basic variables, is nondegenerate, then these basic variables form *the basis* designated as B.

In the given case, the initial, primary variables are $\{x_i, z_{i,j} : i = 1,\ldots,n;\ j = 1,\ldots,m\}$. Variables v_i, u_i and t are complementary, artificial. They are introduced for the purpose of going over from Equations (2), (3) and (5) to Equations (6) to (8).

The variable integrality requirement is ensured by the specific rules of selecting basic variables, which are determined as follows:

The number of variables x_i constituting the basis does not exceed k. If it amounts to k, then variable t equals to 0. If it is less than k, then variable t is above zero.

The variables $\{x_i,\ i = 1,\dots,n\}$ and $\{v_i,\ i = 1,\dots,n\}$ are related univalently: if x_i, = 1, then v_i =0, and vice versa.

Of nm variables $\{z_{i,j} : i = 1,\dots,n;\ j = 1,\dots,m\}$, the basis is constituted of precisely m variables. For the fixed j there is just one i, for which $z_{i,j} > 0$. The corresponding $z_{i,j}$ is a basic variable.

Finally, all the variables u_i , $i = 1,\dots,n$ are included in the basis.

Both the simplex method and the modified simplex method involve the following steps:

1 Choosing the initial basis,

2 Reviewing non-basic vectors and finding such a vector, which, when included in the basis, will increase the target function value.

3 Determining the basic vector to be excluded from the basis.

4 Changing the basis in accordance with the results of Steps 2 and 3.

Steps 2 to 4 shall be repeated until the required vector is found at Step 2.

The specifics of this problem, namely, the restrictions (9) and the abovementioned features, allow us to optimize the implementation of the above-listed steps. Let us elaborate on this in greater detail.

2.3.1 Choosing the initial basis

The number of restrictions of the problem is equal to $2n + m + 1$, which corresponds to the number of basic variables. As the initial basis (initial solution), we can take the following. We can choose a random number of k points of deployment (basing), $k < n$, i.e. k unit values of variables $\{x_i\}$. The corresponding v_i values will be zeros, the other v_i – units, i.e. basic.

Then for each client we randomly choose such a base from the selected points of deployment that will render services for the given client. Formally it means that for each $j = 1,\dots,m$ a certain i is chosen from the number of basic variables, assuming $z_{i,j} = 1$. The other variables $z_{i,j}$ under the fixed j shall be deemed as zeros. Therefore, the number of basic variables $z_{i,j}$ becomes equal to m .

All the variables $\{u_i : i = 1,\dots,n\}$ and variable t are declared to be basic.

Thus, we have selected the required number of $2n + m + 1$variables for the basis.

The above-described rule for selection of variables to be included in the number of basic variables guarantees that the corresponding vectors of Matrix *A* are linearly independent, so the resultant Matrix *B* is nondegenerate, i.e. the *basis*. The basic variables values vector β can be found using the formula

$$\beta = B^{-1} d. \tag{11}$$

2.3.2 Choosing the non-basic vector to be introduced into the basis

Candidates for inclusion in the basis are vectors corresponding to the main variables $\{x_i, z_{i,j} : i = 1, \ldots, n;\ j = 1, \ldots, m\}$ and variables $\{v_1, \ldots, v_n\}$ that are not included in the basis. Now let us introduce Matrix *N* composed of all non-basic vectors. Assumingly, *cb* and *cn* are the vectors of cost coefficients f^T, c^T in the formula (7) for basic and non-basic variables. Then a *relative evaluations vector* is formulated as follows:

$$rc = cb^T B^{-1} N - cn^T. \tag{12}$$

If the vector does not contain positive components, then the current basis is the optimum, and the optimal values of basic variables xb are determined using the formula (11).

If the vector contains positive components, the basis may be improved. Then, among all the positive components, we find the biggest corresponding non-basic variable, which is to be included into the basis. Let us designate the number of this component as p.

2.3.3 Choosing the vector to be excluded from the basis

This vector is found by determining which vector numbered *p* is included into the basis. Let us consider the possible options.

1 Variable $z_{i,j}$ is included in the basis so that x_i must be a basic variable. Then we find the one non-zero variable $z_{i,\eta}$. This variable is *excluded* from the basis.

2 Variable $z_{i,j}$ is included into the basis so that x_i is not a basic variable. In such a case, one must also include variable x_i into the basis. See the next item for the sequence of operations.

3 Non-basic variable x_p is included in the basis. Assumingly, $\tilde{N}$ is a matrix composed of non-basic columns $\{x_i\}$, $\alpha^{(j)} = \left(\alpha_1^{(j)}, \ldots, \alpha_\theta^{(j)}\right)^T$ is Column j of Matrix $B^{-1}\tilde{N}$. Then we find such a number q that

$$x_p = \min\left\{\frac{\beta_\eta}{\alpha_{\eta,p}}\right\} = \frac{\beta_q}{\alpha_{q,p}}, \tag{13}$$

where the minimum suits all the η values, for which $\alpha_{\eta,p} > 0$.

The corresponding basic variable x_q and vector $\alpha^{(q)}$ are excluded from the basis. After that all the basic variables $z_{q,j}$ are replaced with $z_{p,j}$.

2.3.4 Recalculation of the basic matrix

After the basis modification we must recalculate the matrix, which is opposite the basis. Normally, a specially designed recalculation procedure is used. However, there is no need for that since the labour matrix conversion operation is no problem given the present-day level of computer technology development.

Note: The above-described algorithm does not provide for the exclusion of the basic variable x_q from the basis without the inclusion of the other variable x_p. Consequently, in the equation (6) t is always equal to 0. Therefore, one must solve problems for various k and choose the optimal value.

Task Validation Using a Numerical Example

Now please consider a numerical example. Let us assume the following initial conditions.

A. There is no restriction on the maximum number of clients served.

B. The number of points of potential basing (deployment) $n = 9$.

C. The costs f_i are represented by the vector:

$$f = (5 \quad 4 \quad 5 \quad 4 \quad 5 \quad 4 \quad 3 \quad 5 \quad 4).$$

D. The number of clients (points of service) $m = 15$.

The numerical values (reduced) of the costs $c_{i,j}$ are shown in Table 1.

Table 1: Costs $\{c_{i,j}\}$

j\i	1	2	3	4	5	6	7	8	9
1	8.6	8.0	2.0	1.0	4.2	4.2	2.2	1.7	0.9
2	2.4	6.3	3.6	0.3	6.0	7.8	4.8	0.7	0.8
3	0.5	5.9	2.9	2.6	4.1	1.5	4.1	9.6	3.8
4	1.7	1.9	1.0	2.3	6.2	0.9	8.4	4.5	5.2
5	2.2	2.3	7.1	8.6	0.0	4.7	4.3	1.8	1.4
6	7.9	2.4	6.7	0.5	3.9	3.3	7.3	4.7	0.7
7	0.8	4.1	0.1	0.6	3.2	6.1	0.8	2.5	9.1
8	1.4	2.7	6.0	0.9	8.5	2.8	8.5	2.8	7.6
9	5.6	3.8	2.9	9.6	2.0	4.4	3.6	8.2	1.0
10	6.3	2.1	7.5	8.8	4.7	2.2	0.9	6.3	6.8
11	4.5	6.0	9.1	4.0	4.3	5.2	1.9	0.1	8.2
12	3.3	3.6	6.1	6.6	4.2	0.1	9.1	2.9	2.7
13	2.8	9.8	7.8	9.5	5.8	7.7	3.1	8.6	8.7
14	8.7	9.0	5.4	0.0	3.4	9.7	3.4	7.1	9.7
15	8.7	9.5	3.9	0.5	9.5	9.0	2.7	9.2	2.2

The costs $c_{i,j}$ in Table 1 include the losses of opportunity of drone service provision due to meteorological conditions at the chosen points of deployment.

The optimal locations (deployments) of *k* bases under various k are given in Table 2. It shows the points of deployment i_η for each *k*, $\eta = 1,\dots,k$, and the value of Criterion *F*, which is calculated using the formula (1).

Table 2: Optimal deployments of *k* bases and the values of Criterion *F*

k	2	3	4	5	6	7	8	9
i_1	3	3	3	3	3	3	1	1
i_2	6	6	5	5	4	4	3	2
i_3		8	6	6	5	5	4	3
i_4			8	7	6	6	5	4
i_5				8	7	7	6	5
i_7					8	8	7	6
i_8						9	8	7
i_9							9	8
i_{10}								9
F	36.1	30.6	29.5	32.7	36.2	40	44	48.5

As follows from the Table, if the maximum number of bases is four or more, one should choose four bases. In such case the costs will be minimal and amount to 29.5. The optimal assignment of clients to the bases in this case is shown in Table 3, as well as the optimal assignment at $k = 5$.

Table 3: Optimal assignment of Clients *j* to Bases *i* when *k* = 4 and *k* = 5

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	F
i , k = 4	3	3	5	5	8	3	3	3	8	6	6	5	6	3	3	29.5
i , k = 5	8	3	5	5	8	3	3	3	8	6	7	5	6	3	3	32.7

3 RESULTS

The authors set the task of optimal deployment of mobile drone units of a commercial company, which has not been previously considered in scientific literature. To solve the problem with regard to the selected conditions, the linear programming method was used in view of the imposed restriction on the maximum number of clients assigned to Base *i*. The numerical example demonstrated the suggested approach to solving of the set task.

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